

Book Review

Gaertner, W.: *Domain Conditions in Social Choice Theory*. XI, 153 pp.
Cambridge University Press, Cambridge, 2001. Hardcover £ 35.00.

Social choice theory is well known for its impossibility results. It became a field of research following Arrow's impossibility theorem, it attracted renewed attention after Sen's paper on the impossibility of a Paretian liberal, and it connected with game theory and incentive analysis through the proof by Gibbard and Satterthwaite of the negative result that all nondictatorial social choice functions must be manipulable if they choose over more than two alternatives.

Impossibility results are classics in social choice, but this field has more to offer. An impossibility result must be the starting point for analysis of what is possible, what can be done, what one can achieve. It is an invitation to temper our initial ambitions, to look for rules that satisfy milder conditions than those which turn out to be mutually incompatible.

Gaertner's book collects a wide array of possibility results based on the idea that social choice rules need not be defined on universal domains, and that different rules may perform satisfactorily if they are only required to aggregate preference combinations that are adequately restricted by conditions defining which ones belong to the domain of the rule, and which ones do not.

The idea of restricted domains is a powerful one, and it is not exclusive to social choice. One could start the study of general-equilibrium theory by emphasizing the fact that equilibria may not exist unless certain conditions on preferences, endowments and production possibilities are satisfied. From this point of view, proofs of existence of equilibrium are preceded by the establishment of domain restrictions, that is, by specifying assumptions on the basic data of economies for which one can guarantee that an equilibrium is well defined. Of course, discussing whether these assumptions are appropriate, whether they capture some relevant features of the economies that we are interested in modelling, whether one may expect to see them satisfied in other cases, are all important matters of interpretation and analysis that must follow the purely formal activity of defining the domain of application of equilibrium theory.

Generally, any theoretical model must establish the bounds between the combinations of parameters or functional forms that fall within its limits of application and those that are beyond. What may be different in the study of domains in social choice (as compared to other fields of economic analysis), is the variety and the apparent lack of connection between the different proposals that one may find in the literature. I would like to stress that we are facing here a very heterogeneous set of conditions and results, and that Gaertner's objective to bring them all together is not an easy task.

The book is structured in seven chapters. Chapters 1 and 7 provide useful introductory material and concluding comments, respectively. Chapter 2 presents two basic impossibility theorems for unrestricted domains, due to Arrow and to Gibbard and Satterthwaite. These impossibilities are to be contrasted with the possibility results which may be obtained under appropriate domain restrictions, like those presented in the core of the book, chaps. 3 to 6.

Chapter 3 provides an extensive account of domains under which some of the difficulties that are unavoidable in Arrow's framework can be overcome. It is entitled "The existence of collective choice rules under exclusion conditions for finite sets of discrete alternatives." By decomposing the title into different pieces, we can see that it announces well what is covered in chap. 3, and also, by contrast, what will be the topics of the remaining chapters. The reference to "exclusion conditions" means that, in chap. 3, admissible domains are those that only contain certain admissible subsets of preferences. If some preferences are allowed, then others are not. The most salient domain restriction in this class is single-peakedness. Other domain restrictions do not exclude any combination of preferences, but are based on conditions over the distribution of individuals holding each type of preference: these are studied in chap. 5, on "Restrictions on the Distribution of Individuals' Preferences." Again, chap. 3 concentrates on the case with a "finite set of discrete alternatives." In contrast, chap. 6 is devoted to the analysis of "The existence of Social Choice Rules in n -dimensional Continuous Space." Finally, chap. 3 analyzes the "existence of collective choice rules," much in the tradition of Arrow, by concentrating on domains under which some particular rules, especially that of majority, satisfy different consistency conditions. Chapter 4 studies again the possibility of satisfying the Arrowian conditions, as well as others, like strategy-proofness or stability. But the emphasis there is different: what is sought after is no longer a domain that "works" for a given aggregation rule, but rather a characterization of the domains that allow for some rule, any rule, to be defined on them while respecting some set of desirable conditions.

One way to characterize the book is to think of chap. 3 as canonical, referring to a given set of properties that can be guaranteed under a certain type of restrictions by a specific (class of) rule(s). The rest can be considered as evidence that a similar analysis can be carried out with respect to different questions (chap. 4), different kinds of domain restrictions (chap. 5) or different formal setups (chap. 6).

The author had to encompass a very large amount of results, coming from very diverse sources, and using a wide variety of techniques. He has managed to do this beautifully. However, this does not mean that the book is easy reading. The book is probably not one that a non-specialist will read cover-to-cover. But it is definitely a book to know by many, as a reference book, and also as commentary on the importance of the subject. Structuring this material and making it available to others in a tractable way is already an important feat. This alone would make the book a must for people who want to get, in a single shot, the major results obtained by many researchers over more than fifty years. However, the book contains more than a collection of results. Wulf Gaertner is an accomplished social-choice theorist who has made important contributions to the field, and this allows him not to be shy when evaluating the different lines of work that he describes. His discussion of some polemical issues in chap. 6 are an example that this is a book with opinions, much more than a catalog of results.

The book devotes twenty-two of its 153 pages to references and indexes. This gives a measure of the author's scholarship and is definitely a good starting point. However, even a well documented work like this one has its shortcomings. For reasons having to do with my final comments below, I have missed a more thorough treatment of the literature in political science on k -dimensional spaces with saturated preferences (McKelvey and Schofield are mentioned, but not much is said about their work and that of others). The treatment of Grandmont's notion of intermediate preferences could have been continued with references to follow-ups of his seminal idea, like Caplin and Nalebuff's 66% majority rule. Finally, I have missed references to extensions of single-peakedness to more general frameworks, in particular Demange's notion of single-peakedness on a tree.

None of this detracts from the book's importance. In fact, the relevance of the topic may be one of its adversaries, because the area is still active, new ideas keep being produced along different directions, and there is no hope, luckily, that the conditions described here will be the only ones relevant in a few years' time.

Let me just hint at what I believe are promising directions. It is still the case that the most important domain restriction to be known is that of

single-peakedness. Another important domain restriction, of a different character, is intermediateness. Both arise quite naturally in simple enough contexts where one can speak naturally of some betweenness relations, be it among alternatives (single-peakedness), or among agents (intermediate preferences). They have been used extensively in classical applications, and more recently in the literature on political economy. However, they are based on a restrictive version of betweenness that may extend to much more complex networks of relationships among agents and/or among alternatives. Hence, I hope that the rising literature on economic and social networks, coupled with the accumulated knowledge of social-choice theory, will lead to the discovery of new and attractive domain restrictions.

The best success of this book would be to contribute to new developments in its field that would make it age. But then, this is what second editions are for.

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